IMPROV OF A SINGLE-UNIT REPAIRABLE SYSTEM WITHPING UNIT

تطويـــــر الصلاحية لنظام بوحـــــدة واحدة أساسية وأخـــرى مساعدة لها

د. انتصار عبد الله الصويعــــــي الســـــائح ـ المعهد العالي للعلوم والتقنية العزيزية.

الملخص:

تتناول هـــــذه الورقـــــة تحليل النظام القابل للإصـــــلاح لوحدة واحدة مع وحدة مساعدة لها ، الوحدة مفـــردة رئيسية والأخـــــرى تساعد الرئيسية، في البداية تعمل كلتا الوحدتين معا بحيث يعتمد عمل الوحدة الرئيسية على الوحدة المســـــاعدة إذا كانت فعّـــالة ، وإلا فإنها مستقلة عنها مع زيادة معدل الفشل للوحدة المساعدة ، ويتم الفصل بينهم اذا كانت الوحدة المساعدة عاطلة عن العمل تماما ولنفترض أن الفشل والاصلاح والتوصيل والفصل بين الوحدتين متغيرات عشوائية مستقلة عشوائيا لكل منها توزيع معين ، والنظام يعمل بثلاث أوضاع مختلفة (عادي , فشل جزئي , فشل كامل للوحدة الرئيسية والوحدة المساعدة).، حيث يفشل النظام عندما تفشل كلتا الوحدتين فشل كامل ونفترض أن معدل الفشل ومعدل الاصلاح للوحدتين ثوابت.

تم اشتقاق تحويلات لإبلاس للاحتمالات للحالات المختلفة للنظام ، ومن تم الحصول على متوسط الفشل للنظام (MTTF) ، وتم توزيع أوقات الفشل والاصلاح للوحدات بشكل أساسي كما تم تقيم النتائج بمقارنتها بنتائج نظام تم دراسته في السابق والذي يعتبر حالة خاصة منه ، ومن خلال عرض النتائج في جداول وتوضيحها بيانيا يتضح لنا مدى التحسين الذي طرأ على النظام.

IMPROV OF A SINGLE-UNIT REPAIRABLE SYSTEM WITHPING UNIT

Entesar Al- Esayeh
Higher Institute Of Sciences And Technology-Azizia
intesar.ly.2020@gmail.com

ABSTRACT

This paper deals with the analysis of single-unit repairable system with helping unit. The single unit is main and the other is helping. Initially both units operative, the functioning of the main unit is assumed to be dependent on the helping unit if it is operative ,otherwise it is independent with an

increasing failure rate. Assuming that the failure, repair, connecting or disconnecting the min unit from the helping unit times are stochastically independent random variables each having an artistry distribution. The system with three different modes [normal, partial failure, and total failures for the main unite and helping unit]. System failure occurs when both the units fail totally. The failure rate and failed repair rate of a unit are constants. Laplace transforms of the various state probabilities have been derived and then reliability is obtained by the inversion process. Moreover, an important parameter of reliability, i.e. MTTF (mean time to failure) . The failure times of operating/spare units and repair time of failed units are exponential distributed. Certain important result has been evaluated as special cases. Also, few graphical illustrations are also given at the end to high-light the important results.

Introduction And Description Of The System

Introduction of redundancy, are some of the well-known methods by which the reliability of system can be improved. The single-unit repairable system with helping unit redundant systems have been extensively studied by several authors in the past. earlier research worker[4] have studied the reliability cost function analysis of single-unit repairable system with helping unit. The single unit is main and the other is helping , The system with two different modes [normal, and total failures for the main unite and helping unit].

This paper deals with a single unit repairable system with helping unit. The system consists of two units, one is main and the helping unit has three modes-normal mode, partial failure mode and total failure mode. The purpose of using the helping unit is to increase the life time and decrease risk of the system . The main unit can function without the helping unit when the helping unit has failed .The main unit is not allowed to work without the helping unit if it is in a normal mode. A single repair facility is available with the system to repair the main unit from partial failure mode or total failure mode or the helping unit from the partial failure mode or total failure mode, priority is given to the repair of the main unit. After repair of a failed unit it it becomes like a new one. Laplace transforms of the various state probabilities have been derived and then reliability is obtained by the inversion process. Moreover, an important parameter of reliability, i.e. MTTF (mean time to failure), system availability and steady-state availability are derived.

Redundancy plays a pivotal role in improving system reliability.

The paper presents mathematical system representing single-unit repairable system with helping unit with three modes of each unit normal (N), partial failure(Pr) and total failure(Tr) .The mean-time to system failure, reliability to system and steady-state availability are obtained using the transitions of the Markov process to the up states and taking Laplace transform of the various state probabilities have been derived and then reliability is obtained by the inversion process. The failure times of operating/spare units and repair time of failed units are exponential distributed. The effects of additional helping unit on the system performance are shown graphically.

In this system The following assumptions and notations are used to analyze the system

1. The system consists of two unit ,one is main and the other is helping.
2. A main unit has three possible modes-normal, partial failure and total failure, also a helping unit has three possible modes-normal, partial failure and total failure.
3. The functioning of the main unit is assumed to be dependent on the helping unit if it is operative, otherwise it is independent with an increasing failure rate when the helping unit has failed.
4. The main unit is allowed to work without the helping unit if it is in a normal mode and partial failure.
5. There is a single repair facility with the system to repair the totally failed unit (main or helping)or the partially failed main unit.
6. The main unit can function without the helping unit when the helping unit has failed.
7. The main unit is allowed to work without the helping unit if it is in a normal and partial to decrease the risk of failing the main unit.
8. The main unit can fail either partially or totally, while the helping unit can fail only totally.
9. Priority is given to repair of the main unit.
10. After repair of a failed unit it becomes like a new one.
11. Partial failure and total failure of the main unit, partial failure and total failure of the helping unit, partial repair and total repair or the main

unit and total repair of the helping unit, connect or disconnect the main unit from the helping unit times are stochastically independent random variables cache having an arbitrary distribution.

12. All random variables are mutually independent.

System used for the states:

o_1 the main unit is operative.

o_2 the helping unit is operative.

p_{1r} the main unit fails partially and is under repair.

p_{2r} the helping unit fails partially and is under repair.

T_{1r} the main unit fails totally and is under repair.

T_{2r} the helping unit fails totally and is under repair.

T_{1wr} the main unit fails totally and is waiting for repair.

T_{2wr} the helping unit fails totally and is waiting for repair.

c connection the main unit from the helping.

d disconnection the main unit from the helping unit.

α constant failure rate of a main unit.

β constant failure rate of a helping unit.

λ_1 constant repair partial failures rate of a main from state j to state.

λ_2 constant repair partial failures rate of a helping unit from state j to state.

μ_1 constant repair total failures rate of a main from state j to state.

μ_2 constant repair total failures rate of a helping unit from state j to state.

Notations And States Of The System

$p_j(t)$ probability that at time t, the system is in Sj state (j = 0, 1, 2… 10),

s Laplace transform variable,

F(s) Laplace transform of F (t).

Considering these symbols, the system may be in one of the following states:

$s_0 \equiv (o_1, o_2, c)$, $s_1 \equiv (p_{1r}, o_2, c)$, $s_2 \equiv (o_1, p_{2r}, c)$, $s_3 \equiv (T_{1r}, p_{2r}, c)$, $s_4 \equiv (p_{1r}, T_{2r}, d)$,

$s_5 \equiv (o_1, T_{2r}, d)$, $s_6 \equiv (T_{1r}, o_2, c)$, $s_7 \equiv (p_{1r}, T_{2wr}, c)$, $s_8 \equiv (T_{2wr}, p_{2r}, c)$, $s_9 \equiv (T_{1wr}, T_{2r}, d)$,

$s_{10} \equiv (T_{1r}, T_{2wr}, d)$.

Up states: $s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8$.Down states : s_9, s_{10} .

State and possible transitions between them are shown in Figure 1.

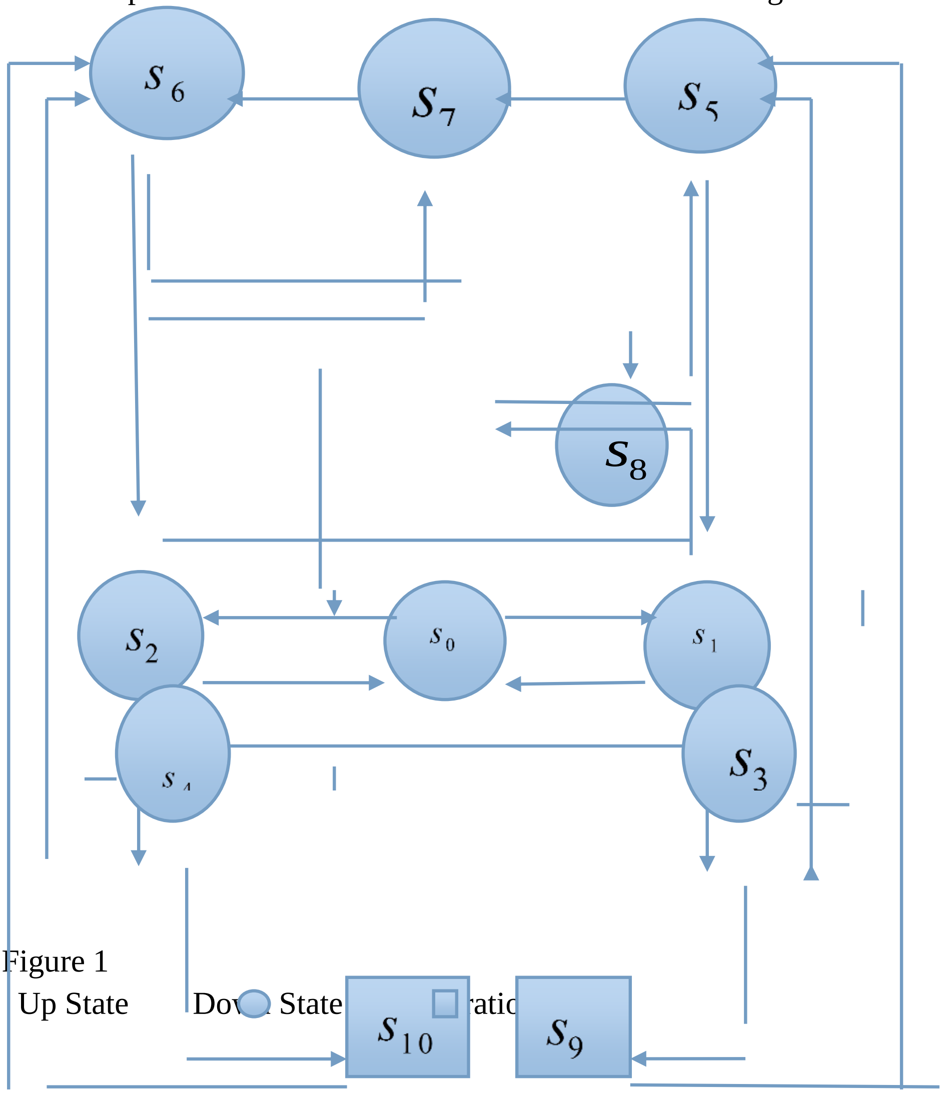

Figure 1

Using the transitions of the Markov process to the up states of the system. let $p_i(t), i = 0,1,\ldots\ldots,10$ be the probability that the system is in state s_i at time t. The infinitesimal generator of the Markove process is given below

$$\begin{pmatrix} (s+\alpha+\beta) & -\lambda_1 & -\lambda_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\alpha & (s+2\lambda_1+2\beta) & -\lambda_2 & 0 & 0 & -\mu_2 & 0 & 0 & 0 & 0 \\ -\beta & \lambda_1 & (s+2\alpha+2\lambda_2) & 0 & 0 & 0 & -\mu_1 & 0 & 0 & 0 \\ 0 & -\beta & 0 & (s+\mu_1) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\alpha & 0 & (s+\mu_2) & 0 & 0 & 0 & 0 & -\mu_2 \\ 0 & 0 & 0 & -\mu_1 & 0 & (s+\alpha+\mu_2) & 0 & 0 & -\mu_1 & 0 \\ 0 & 0 & 0 & 0 & -\mu_2 & 0 & (s+\mu_1+\beta) & -\mu_2 & 0 & 0 \\ 0 & 0 & -\alpha & 0 & 0 & -\alpha & 0 & (s+\mu_2) & 0 & 0 \\ 0 & -\beta & 0 & 0 & 0 & 0 & -\beta & 0 & (s+\mu_1) & 0 \\ 0 & 0 & 0 & 0 & -\alpha & 0 & 0 & 0 & 0 & (s+\mu_2) \end{pmatrix} \quad (1)$$

We assume that initially both the units are operable and obtain the measures of system performance

System Reliability

The system reliability R(t) is the probability of failure-free operation of the system in (0,t]. To derive an expression for the reliability of the system, we restrict the transitions of the Markov process to the up states, viz. $s_0 - s_8$ .Using the infinitesimal generator given in (1),pertaining to these states and standard probabilistic arguments, we derive the following differential equations:

$$P_0'(t) = -(\alpha+\beta)P_0(t) + \lambda_1 P_1(t) + \lambda_2 P_2(t),$$

$$P_1'(t) = -(2\beta+2\lambda_1)P_1(t) + \alpha P_0(t) + \lambda_2 P_2(t) + \mu_2 P_5(t),$$

$$P_2'(t) = -(2\alpha+2\lambda_2)P_2(t) + \beta P_0(t) + \lambda_1 P_1(t) + \mu_1 P_6(t),$$

$$P_3'(t) = -(\beta+\mu_1)P_3(t) + \beta P_1(t),$$

$$P_4'(t) = -(\alpha+\mu_2)P_4(t) + \alpha P_2(t),$$

$$P_5'(t) = -(\alpha+\mu_2)P_5(t) + \mu_1 P_3(t) + \mu_1 P_8(t),$$

$$P_6'(t) = -(\beta+\mu_1)P_5(t) + \mu_2 P_4(t) + \mu_2 P_7(t),$$

$$P_7'(t) = -(\mu_2)P_7(t) + \alpha P_2(t) + \alpha P_5(t),$$

$$P_8'(t) = -(\mu_1)P_8(t) + \beta P_1(t) + \beta P_6(t).$$

(2)

Let $L_i(s)$ be the Laplace transform of $p_i(t) = 0,1,\ldots\ldots,8$ .Taking Laplace transform on both the sides of the differential equations (2)and using the initial conditions $p_0(0)=1, p_i(0)=0$,where $i=1,2,........,9$,solving for $L_i(s), i=0,1,.....,8$,we get

$$(s+\alpha+\beta)P_0(s)-\lambda_1 P_1(s)-\lambda_2 P_2(s)=1,$$
$$(s+2\beta+2\lambda_1)P_1(s)-\alpha P_0(s)-\lambda_2 P_2(s)-\mu_2 P_5(s)=0,$$
$$(s+2\alpha+2\lambda_2)P_2(s)-\beta P_0(s)-\lambda_1 P_1(s)-\mu_1 P_6(s)=0,$$
$$(s+\beta+\mu_1)P_3(s)-\beta P_1(s)=0,$$
$$(s+\alpha+\mu_2)P_4(s)-\alpha P_2(s)=0,$$
$$(s+\alpha+\mu_2)P_5(s)-\mu_1 P_3(s)-\mu_1 P_8(s)=0,$$
$$(s+\beta+\mu_1)P_5(s)-\mu_2 P_4(s)-\mu_2 P_7(s)=0,$$
$$(s+\mu_2)P_7(s)-\alpha P_2(s)-\alpha P_5(s)=0,$$
$$(s+\mu_1)P_8(s)-\beta P_1(s)-\beta P_6(s)=0.$$

(3)

And inverting, we get $p_i(t), i=0,1,....,8$.Then the system reliability is given by

$$R(t)=P_0(t)+P_1(t)+P_2(t)+P_3(t)+P_4(t)+P_5(t)+P_6(t)+P_7(t)+P(t)_8$$
$$=\sum_{i=1}^{9}\frac{Q_1 e^{s_i t}}{\prod_{r=1,r\neq i}^{9}}$$

(4)

Where

$$Q_1=[-ER-GR-E\alpha(s_i+\mu_1)-G\beta(s_i+\mu_2)+2\beta\mu_1 R(-A+\alpha(\beta(s_i+2\beta)$$
$$-(\alpha-2\beta)\lambda_1)\mu_2)+\alpha(s_i+\mu_1)(DB-2\alpha^2\beta\mu_1^2\mu_2+$$
$$\beta\mu_1(2D\alpha(s_i+2\alpha)+2D(2\alpha+\beta)\lambda_2+(\beta(-D+s_i\alpha+2\alpha\beta)$$

$-\alpha\left(\alpha-2\beta\right)\lambda_1)\mu_2))+2R\alpha\mu_2(-F(\alpha-2\beta)\lambda_1+\beta(F(s_i+2\beta)$

$+\mu_1(\alpha(s_i+2\alpha)+(2\alpha+\beta)\lambda_2-2\beta\mu_2)))$

$+R(DF(H+2\alpha\beta\mu_1^2\mu_2(-\lambda_2+2\mu_2)-\mu_1\mu_2(\alpha s_i(F+R)+2\beta D(s_i+2\alpha)$

$+\alpha\beta(2F+H)+2\beta(\alpha R+2D\lambda_2)+2\alpha\lambda_1(F+R-\beta\mu_2)))$

$+\beta(s_i+\mu_2)(-AF+B\alpha\mu_2+\alpha\mu_2(-F(\alpha-2\beta)\lambda_1+\beta(F(s_i+2\beta)$

$+\mu_1(\alpha(s_i+2\alpha)+(2\alpha+\beta)\lambda_2-\beta\mu_2))))]$

(5)

and $s_1, s_2, ..., s_6$ are roots of the equation (3)

Mean Time To System Failure

The steady-state reliability of the system is given by

$R(t)=P_0(s)+P_1(s)+P_2(s)+P_3(s)+P_4(s)+P_5(s)+P_6(s)+P_7(s)+P(s)_8$

$=[-ER-GR-E\alpha\left(s+\mu_1\right)-G\beta\left(s+\mu_2\right)+2\beta\mu_1R(-A+\alpha(\beta(s+2\beta)$

$-(\alpha-2\beta)\lambda_1)\mu_2)+\alpha\left(s+\mu_1\right)(DB-2\alpha^2\beta\mu_1^2\mu_2+$

$\beta\mu_1(2D\alpha\left(s+2\alpha\right)+2D(2\alpha+\beta)\lambda_2+(\beta(-D+s\alpha+2\alpha\beta)$

$-\alpha\left(\alpha-2\beta\right)\lambda_1)\mu_2))+2R\alpha\mu_2(-F(\alpha-2\beta)\lambda_1+\beta(F(s+2\beta)$

$+\mu_1(\alpha(s+2\alpha)+(2\alpha+\beta)\lambda_2-2\beta\mu_2)))$

$+R(DF(H+2\alpha\beta\mu_1^2\mu_2(-\lambda_2+2\mu_2)-\mu_1\mu_2(\alpha s(F+R)+2\beta D(s+2\alpha)$

$+\alpha\beta(2F+H)+2\beta(\alpha R+2D\lambda_2)+2\alpha\lambda_1(F+R-\beta\mu_2)))$

$+\beta(s+\mu_2)(-AF+B\alpha\mu_2+\alpha\mu_2(-F(\alpha-2\beta)\lambda_1+\beta(F(s+2\beta)$

$+\mu_1(\alpha(s+2\alpha)+(2\alpha+\beta)\lambda_2-\beta\mu_2))))/\prod_{r=1}^{9}\left(s-s_r\right)]$

(6)

The mean time to failure of the system is given by

$MTTF=-E_0R_0-G_0R_0-E_0\alpha\mu_1-G_0\beta\mu_2+$

$2\beta\mu_1R_0(-A_0+\alpha(2\beta^2-(\alpha-2\beta)\lambda_1)\mu_2)$

$+\alpha\mu_1(D_0B_0-2\alpha^2\beta\mu_1^2\mu_2+\beta\mu_1(4D_0\alpha^2+2D_0(2\alpha+\beta)\lambda_2+(\beta(-D_0+2\alpha\beta)$

$-$

$$\alpha(\alpha-2\beta)\lambda_1)\mu_2))+2R_0\alpha\mu_2(-F_0(\alpha-2\beta)\lambda_1+\beta(F_0 2\beta$$
$$+\mu_1(\alpha 2\alpha+(2\alpha+\beta)\lambda_2-2\beta\mu_2)))$$
$$+R_0(D_0F_0(H_0+2\alpha\beta\mu_1^2\mu_2(-\lambda_2+2\mu_2)-\mu_1\mu_2(4\alpha\beta D_0$$
$$+\alpha\beta(2F_0+H_0)+2\beta(\alpha R_0+2D_0\lambda_2)+2\alpha\lambda_1(F_0+R_0-\beta\mu_2)))$$
$$+\beta\mu_2(-A_0F_0+B_0\alpha\mu_2+\alpha\mu_2(-F_0(\alpha-2\beta)\lambda_1+\beta(2\beta F_0$$
$$+\mu_1(\alpha^2 2+(2\alpha+\beta)\lambda_2-\beta\mu_2))))/\prod_{r=1}^{9}(-s_r)$$

(7)

Where $\prod_{r=1}^{9}(-s_r)$ are the roots of the equation(2,3)and

$$A_i=(2\alpha^2\mu_1\mu_2-(\alpha(s_i+2\alpha)+(2\alpha+\beta)\lambda_2)D_i),$$
$$B_i=(-\beta^2\mu_1\mu_2+(\beta(s_i+2\beta)-(\alpha-2\beta)\lambda_1)F_i),$$
$$C_i=(-s_i\alpha\beta-2\alpha\beta^2+\alpha^2\lambda_1-2\alpha\beta\lambda_1+\alpha^2\mu_1-\beta D_i),$$
$$D_i=(s_i+\beta+\mu_1)(s_i+\mu_2),$$
$$E_i=(\alpha^2\beta\mu_1^2\mu_2-\beta\mu_1\mu_2C_i-D_iB_i),$$
$$F_i=(s_i+\mu_1)(s_i+\alpha+\mu_2)$$
$$G_i=(\alpha\beta\mu_1\mu_2(\alpha(s_i+2\alpha)+(2\alpha+\beta)\lambda_2-2\beta\mu_2)+F_iA_i),$$
$$R_i=(s_i+\mu_1)(s_i+\mu_2).$$

(8)

since $A_i=A_0\Rightarrow s_i=0, A_i=A\Rightarrow s_i=s.$

Special Case

When partial failure is not allowed

Considering these symbols, the system may be in one of the following states:

$s_0\equiv(o_1,o_2,c)$, $s_1\equiv(P_{1r},o_2,c)$, $s_2\equiv(T_{1r},o_{2r},d)$, $s_3\equiv(T_{2r},o_{1r},d)$, $s_4\equiv(o_{1r},T_{2r},c)$,

$s_5\equiv(o_1,o_{2r},d)$, $s_6\equiv(P_{1r},T_{2r},c)$, $s_7\equiv(T_{1r},T_{2wr},d)$.

Up states: s_0,s_1,s_4,s_6 .Down states : s_2,s_3,s_5,s_7 .

State and possible transitions between them are shown in Figure

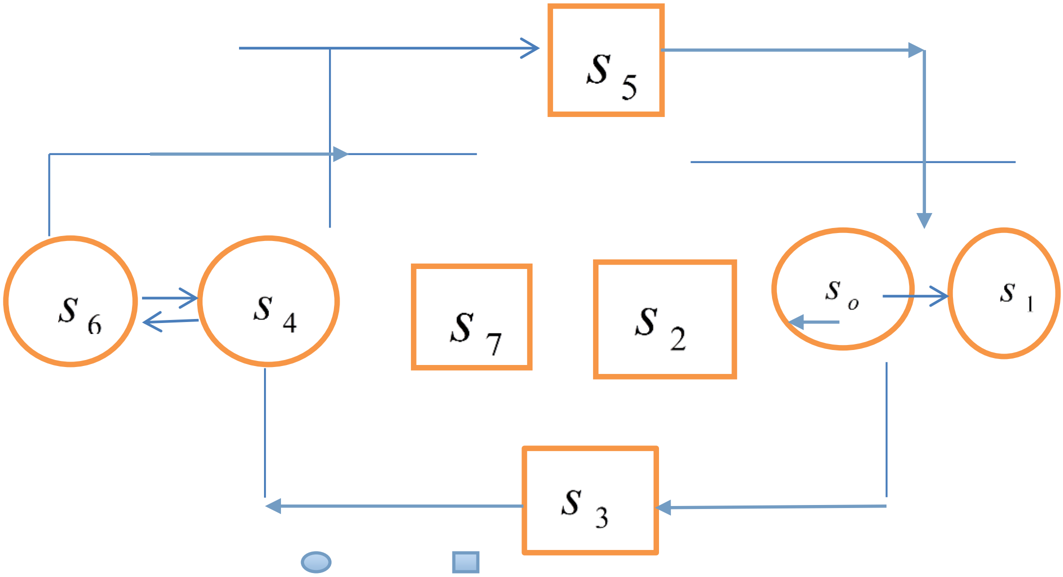

Figure 2

Up State Down State . Regeneration Point

Let $P_i(t), i = 0,1,2,3,4,5,6,7$ be the probability that the system is state S_i at time t.

.The mean-time to system failure, reliability to system are obtained using the transitions of the Markov process to the up states and taking Laplace transform of the various state probabilities have been derived and then reliability is obtained by the inversion process. The failure times of operating/spare units and repair time of failed units are exponential distributed. The effects of additional helping unit on the system performance are shown graphically.

The mean time to failure of the system (2) is given by:

$$MTTF=\frac{\alpha^2\lambda_1+\alpha\lambda_1^2}{\alpha^2\lambda_1^2+\alpha\beta\lambda_1^2}.$$

Numerical Example

In the special cases, let the constant failure rate of a main unit $\alpha=(0,0.3]$.

,the constant failure rate of a helping unit $\beta=0.05$,the constant repair partial failures rate of a main from state j to state $\lambda_1=0.9$,the constant repair partial failures rate of a helping unit from state j to state $\lambda_2 = 0.9$,the constant repair total failures rate of a main from state j to state $\mu_1 = 0.9$,and the constant repair total failures rate of a helping unit from state j to state $\mu_2 = 0.9$

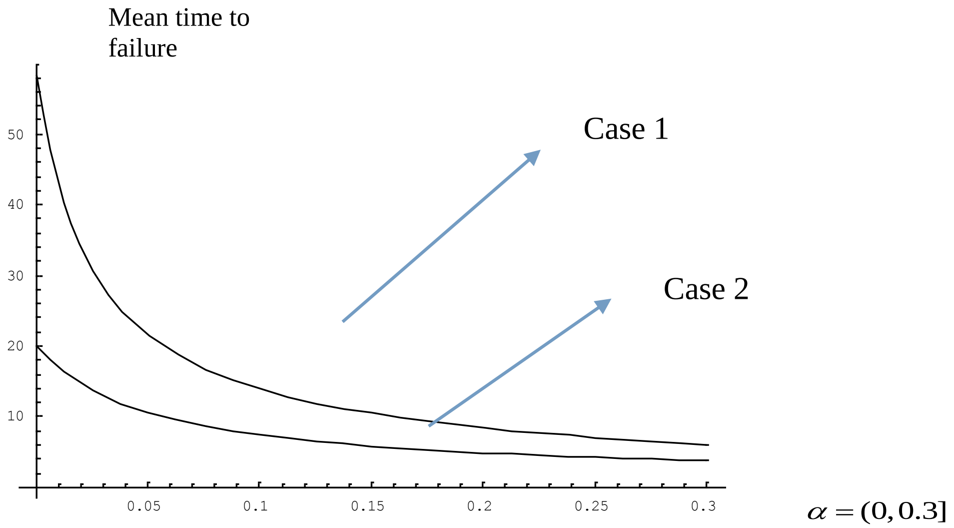

Figure 3:Relation Between The constant failure rate of a main unit and Mean Time To System failure for cases.

α	MTSF case 1	MTSF case 2
0.05	**17.5906**	**10.5556**
0.06	**15.5726**	**9.69697**
0.07	**13.9772**	**8.98148**
0.08	**12.6835**	**8.37607**
0.09	**11.6129**	**7.85714**
0.1	**10.7117**	**7.40741**
0.2	**6.04964**	**4.88889**
0.3	**4.18607**	**3.80952**

Table 1:Relation Between α an the MTSF

λ_1	MTSF case 1	MTSF case 2
0.1	**17.2952**	**13.5714**
0.2	**14.0989**	**10.3571**
0.3	**13.0334**	**9.28571**
0.4	**12.5007**	**8.7500**

0.5	**12.1811**	**8.42857**
0.6	**11.968**	**8.21429**
0.7	**11.8158**	**8.06122**
0.8	**11.6129**	**7.94643**

Table 2:Relation Between λ_1 an the MTSF

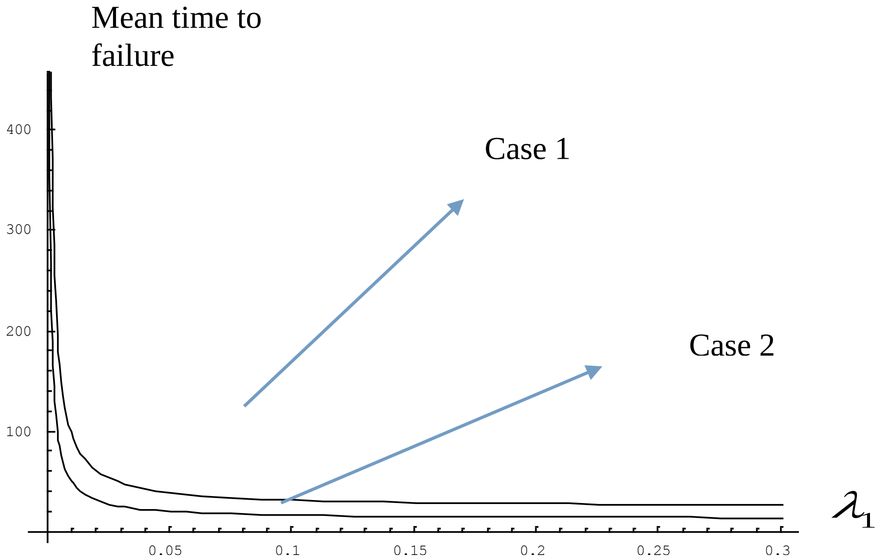

Figure 4:Relation Between The constant repair partial failures rate of a main unit and Mean Time To System failure for cases.

Conclusion

Figures (3,4) show that the present of additional helping unit for system when the operative state go to the partial failure lead to improve the values of the mean time to system failure ,also we conclude that the mean time to first system is greater than the second system for all α value and the mean time to system failure for first system is increases when λ_1, α is large .Table (1) compute the MTTF for two system and profit of the first system from rate α is large. We note from table (2) MTTF increase of the first system when λ_1 is large .The curves of MTTF show that is failure rates(λ_1, α) increase, MTTF decreases. This Conclusion show that the first system is better than the second system.

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